NUMERICAL ANALYSIS

Chapter:- 1

Elementary Mathematical Concepts and Errors:-

45. Consider the function f(x) = e^{-x} and its Taylor approximation g(x) of degree 3. For $x = \frac{1}{3}$, g(x) is

(1) Positive and less than 1
(2) Negative and less than -2
(3) Positive and greater than 1
(4) Less than 1 but greater than 0.75

[June 2011 (3 Marks)]

43. Let $f(x)=x^2+2x+1$ and the derivative of f at $x=1$ is approximated by using the central difference formula $f'(1)\approx\frac{f(1+h)-f(1-h)}{2h}$ with $h=\frac{1}{2}$.

Then the absolute value of the error in the approximation of $f'(1)$ is equal to

(1) 1 (2) 1/2
(3) 0 (4) 1/12

[June 2016 (3 Marks)]

44. The magnitude of the truncation error for the scheme $f'(x) = Af(x) + Bf(x+h) + Cf(x+2h)$ is equal to

(1) $h^2 f'''(\xi)$ if $A=-\frac{5}{6h}, B = \frac{3}{2h'} C = -\frac{2}{3h}$

(2) $h^2 f'''(\xi)$ if $A=\frac{5}{6h}, B = \frac{3}{2h'} C = \frac{2}{3h}$

(3) $h^2 f''(x)$ if $A=-\frac{5}{6h}, B = \frac{3}{2h'} C = -\frac{2}{3h}$.

(4) $h^2 f''(x)$ if $A=\frac{5}{6h}, B = \frac{3}{2h'} C = \frac{2}{3h}$.

[June 2017 (3 Marks)]

115. Consider a sufficiency smooth function f(x). A formula for estimating its derivative is given by

$$\frac{df}{dx} = \frac{1}{4h}\left[f(x+2h) - f(x-2h)\right] + error\ term$$

where h > 0. Let $f^{(n)}$ denote the nth derivative of f and let ζ be a point between x - 2h and x + 2h. Which of the following expressions for the error term are correct?

(1) $\frac{-f^{(2)}(\zeta)h^3}{2}$ (2) $\frac{-2f^{(3)}(\zeta)h^2}{3}$

(3) $-f^{(1)}(\zeta)h$ (4) $\frac{-f^{(4)}(\zeta)h^4}{12}$

[June 2013 (4.75 Marks)]

93. Let $f(x) = e^x$ be approximated by Taylor's polynominal of degree n at the point $x = \frac{1}{2}$ and on the entire inteval [0,1). If the absolute error in this approximation does not exceed 10^{-2} then the value of n should be taken as

(1) 0 (2) 1
(3) 2 (4) 3

[Dec. 2013 (4.75 Marks)]

102. Let $\in C^3([x_{-1,} x_1])$ *where*

$x_{-1} = x_0 - h, x_1 = x_0 + h$ with h >0,

$f(x_0) = f_0, f(x_j) = f_j$ *for* $= -1,1$ *and* $f'(x_0) = f_0'$

Then for some $\xi \in \left(X_{-1,} X_1\right)$ we have

(1) $f'_0 = \frac{f_1 - f_0}{h} - \frac{h^2}{2} f'''(\xi).$

(2) $f'_0 = \frac{f_1 - f_{-1}}{h} - \frac{h^3}{3} f'''(\xi)$

(3) $f'_0 = \frac{f_1 - f_{-1}}{2h} - \frac{h^2}{6} f'''(\xi).$

(4) $f'_0 = \frac{f_1 - f_{-1}}{2h} + \frac{h^3}{6} f'''(\xi).$

[Dec. 2013 (4.75 Marks)]

101. Which of the following approximations for estimating the derivative of a smooth function f at a point x is of order 2(i.e the error term is $O(h^2)$)

(1) $f'(x) \approx \frac{f(x+h) - f(x)}{h}$

(2) $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$

(3) $f'(x) \approx \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$

(4) $f'(x) \approx \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}$

[Dec. 2014 (4.75 Marks)]

93. The values of a, b, c so that the truncation error in the formula

$$\int_{-h}^{h} f(x)dx = ahf(-h) + bhf(0) + ahf(h)$$

$$+ch^2 f'(-h) - ch^2 f'(h)$$

is minimum, are

(1) $a = \frac{7}{15}, b = \frac{16}{15}, c = \frac{1}{15}$

(2) $a = \frac{7}{15}, b = \frac{16}{15}, c = \frac{-1}{15}$

(3) $a = \frac{7}{15}, b = \frac{-16}{15}, c = \frac{1}{15}$

(4) $a = \frac{7}{15}, b = \frac{-16}{15}, c = \frac{-1}{15}$

[June 2019 (4.75 Marks)]

Answer Key

Chapter: - 1. Mathematical Preliminaries and Errors:-

1.	**June 2011**	**Q. 45**	**(1)**
2.	**June 2016**	**Q. 43**	**(3)**
3.	**June 2017**	**Q. 44**	**(*)**
4.	**June 2013**	**Q. 115**	**(2)**
5.	**Dec. 2013**	**Q. 93**	**(4)**
6.	**Dec. 2013**	**Q. 102**	**(3)**
7.	**Dec. 2014**	**Q. 101**	**(2,3,4)**
8.	**June 2019**	**Q. 93**	**(1)**

Chapter:- 2

Solution of Algebraic and Transcendental Equations:-

47. The initial value problem

$$\frac{\partial u}{\partial t}+x\frac{\partial u}{\partial x}=0, \quad 0\le x\le 1, \quad t>0 \quad \text{and}$$

$u(x,0)=2x$ has

(1) a unique solution $u(x,t)$ which $\to\infty$ as $t\to\infty$

(2) more than one solution

(3) a solution which remains bounded as $t\to\infty$

(4) no solution

[June 2014 (3 Marks)]

45. Let $f(x)=ax+100$ for $a\in\mathbb{R}$. then the iteration $x_{n+1}=f(x_n)$ for $n\ge 0$ and $x_0=0$ converges for

(1) $a = 5$ (2) $a = 1$

(3) $a = 0.1$ (4) $a = 10$

[Dec. 2015 (3 Marks)]

43. The values of α and β, such that

$$x_{n+1}=\alpha x_n\left(3-\frac{x_n^2}{a}\right)+\beta x_n\left(1+\frac{a}{x_n^2}\right)$$

has 3^{rd} order convergence to $\sqrt{a}$, are

(1) $\alpha=\frac{3}{8},\beta=\frac{1}{8}$ (2) $\alpha=\frac{1}{8},\beta=\frac{3}{8}$

(3) $\alpha=\frac{2}{8},\beta=\frac{2}{8}$ (4) $\alpha=\frac{1}{4},\beta=\frac{3}{4}$

[Dec. 2016 (4.75 Marks)]

45. The iterative method $x_{n+1}=g(x_n)$ for the solution of $x^2-x-2=0$ converges quadratically in a neighbourhood of the root $x=2$ if $g(x)$ equals

(1) x^2-2 (2) $(x-2)^2-6$

(3) $1+\frac{2}{x}$ (4) $\frac{x^2+2}{2x-1}$

[Dec. 2017 (3 Marks)]

46. Let $x=\xi$ be a solution of $x^4-3x^2+x-10=0$. The rate of convergence for the iterative method $x_{n+1}=10-x_n^4+3x_n^2$ is equal to

(1) 1 (2) 2

(3) 3 (4) 4

[Dec. 2019 (3 Marks)]

98. Consider the iteration function for Newton's method $g(x)=x-\frac{f(x)}{f'(x)}$ and its application to find (approximate) square root of 2, starting with $x_0 = 2$. Consider the first and the second iterate x_1 and x_2, respectively; then

(1) $1.5 < x_1 \le 2$

(2) $1.5 \le x_1 < 2$

(3) $x_1 \le 1.5$; $x_2 \le 1.5$

(4) $x_1 = 1.5$; $x_2 < 1$

[June 2011 (4.75 Marks)]

98. Consider the function $f(x)=x^2-x-2=0$

Let $x = g(x)$, so that any fixed point of $g(x)$ is a solution of (1). Then

(1) $g(x)=x-\frac{x^2-x-2}{m}$, $m\in[-a,a]$ is a possible choice where a is positive constant.

(2) $g(x)=x^2-2, g(x)=1+\frac{2}{x}$ are possible choice

(3) $g(x)=x-\frac{x^2-x-2}{k}, k\ne 0, K\in R$ is a possible choice

(4) $g(x)=x^2-2, g(x)=1+\frac{2}{x}$ are the only possible choice

[June 2012 (4.75 Marks)]

95. Let f be a continuous map from the interval [0, 1] into itself and consider the iteartion $x_{n+1}=f(x_n)$

Which of the following maps will yield a fixed point for f ?

(1) $f(x)=\frac{x^2}{4}$ (2) $f(x)=\frac{x^2}{8}$

(3) $f(x)=\frac{x^2}{16}$ (4) $f(x)=\frac{x^2}{32}$

[Dec. 2012 (4.75 Marks)]

100. Let $f(u)=u^3-u-1$

(1) Starting with the initial guess $u^{(0)}=1.5$, the fixed point iterates of the equation u = g(u), where g(u) = u^3 – 1 converge.

(2) Starting with the initial guess $u^{(0)}=1.5$, the fixed point iterates of the equation $u=\tilde{g}(u)$,

where $\tilde{g}(u)=\sqrt{1+u^3}$ converge.

(3) If u* is a root of the equation *f*(u) = 0 and u* > 1, then u* is a stable fixed point of the equation u = g(u)

(4) *f*(u) = 0 has a root between 1 and 2

[Dec. 2012 (4.75 Marks)]

94. Consider the function $f(x)=\sqrt{2+x}$ for $x\ge -2$ and the iteration $x_{n+1}=f(x_n)$; $n\ge 0$ for $x_0=1$. What are the possible limits of the iteration?

(1) $\sqrt{2+\sqrt{2+\sqrt{2+...}}}$ (2) -1

(3) 2 (4) 1

[June 2014 (4.75 Marks)]

95. Consider the iteration $x_{n+1}=\frac{1}{2}\left(x_n+\frac{2}{x_n}\right) n\ge 0$ for a given $x_0\ne 0$. Then

(1) x_n converges to $\sqrt{2}$ with rate of convergence 1.

(2) x_n converges to $\sqrt{2}$ with rate of convergence 2.

(3) The given iteration is the fixed point iteration for $f(x)=x^2-2$

(4) The given iteration is the Newton's method for $f(x)=x^2-2$

[June 2014 (4.75 Marks)]

98. Let $f:\mathbb{R}\to\mathbb{R}$ be a smooth function with non-vanishing derivative. The Newton's method for finding a root of $f(x)=0$ is the same as

(1) Fixed point iteration for the map

$$g(x) = x-f(x)/f'(x)$$

(2) Forward Euler method with unit step length for the differential equation

$$\frac{dy}{dx}+\frac{f(y)}{f'(y)} = 0$$

(3) Fixed point iteration for $g(x)=x+f(x)$

(4) Fixed point iteration for $g(x)=x-f(x)$

[Dec. 2014 (4.75 Marks)]

101. The iteration $x_{n+1}=\frac{1}{2}\left(x_n+\frac{2}{x_n}\right)$, $n\ge 0$ for a given $x_0\ne 0$ is an instance of

(1) fixed point iteration for $f(x)=x^2-2$

(2) Newton's method for $f(x)=x^2-2$

(3) fixed point iteration for $f(x)=\frac{x^2+2}{2x}$

(4) Newton's method for $f(x)=x^2+2$

[Dec. 2015 (4.75 Marks)]

96. Let $f(x)=\sqrt{x+3}$ for $x\ge -3$. Consider the iteration $x_{n+1}=f(x_n)$, $x_0=0$; $n\ge 0$. The possible limits of the iteration are

(1) -1 (2) 3

(3) 0 (4) $\sqrt{3+\sqrt{3+\sqrt{3+...}}}$

[Dec. 2015 (4.75 Marks)]

92. Let $f:[0,1]\to[0,1]$ be twice continuously differentiable function with a unique fixed point $f(x_*)=x_*$. For a given $x_0\in(0,1)$ consider the iteration $x_{n+1}=f(x_n)$ for $n\ge 0$.

If $L=\max_{x\in[0,1]}|f'(x)|$, then which of the following are true?

(1) If $L<1$, then x_n converges to x_*

(2) x_n converges to x_* provided $L\ge 1$

(3) The error $e_n=x_n-x_*$ satisfies $|e_{n+1}|<L|e_n|$

(4) If $f'(x_*)=0$, then $|e_{n+1}|<C|e_n|^2$ for some $C>0$

[Dec. 2018 (4.75 Marks)]

94. Consider the equation $x^2 + ax + b = 0$ which has two real roots α and β. Then which of the following iteration scheme converges when x_0 is chosen sufficiently close to α?

(1) $x_{n+1} = -\dfrac{ax_n + b}{x_n,}$ if $|\alpha| > |\beta|$

(2) $x_{n+1} = -\dfrac{x_n^2 + b}{a}$ if $|\alpha| > 1$

(3) $x_{n+1} = -\dfrac{b}{x_n + a}$, if $|\alpha| < |\beta|$

(4) $x_{n+1} = -\dfrac{x_n^2 + b}{a}$, if $2|\alpha| < |\alpha + \beta|$

[June 2019 (4.75 Marks)]

Answer Key

Chapter: - 2. Solution of Algebraic and Transcendental Equation:-

1.	**June 2014**	**Q. 47**	**(3)**
2.	**Dec. 2015**	**Q. 45**	**(3)**
3.	**Dec. 2016**	**Q. 43**	**(2)**
4.	**Dec. 2017**	**Q. 45**	**(3)**
5.	**Dec. 2019**	**Q. 46**	**(1)**
6.	**June 2011**	**Q. 98**	**(2,3)**
7.	**June 2012**	**Q. 98**	**(2,3)**
8.	**Dec. 2012**	**Q. 95**	**(1,2,3,4)**
9.	**Dec. 2012**	**Q. 100**	**(4)**
10.	**June 2014**	**Q. 94**	**(1,3)**
11.	**June 2014**	**Q. 95**	**(2,4)**
12.	**Dec. 2014**	**Q. 98**	**(1,2)**
13.	**Dec. 2015**	**Q. 101**	**(2,3)**
14.	**Dec. 2015**	**Q. 96**	**(4)**
15.	**Dec. 2018**	**Q. 92**	**(1,3,4)**
16.	**June 2019**	**Q. 94**	**(1,3,4)**

Chapter:- 3

Interpolation And Approximation:-

26. If the points $x_1, x_2, ..., x_n$ are distinct, then for arbitrary real values $y_1, y_2, ..., y_n$ the degree of the unique interpolating polynomial p(x) such that $p(x_i) = y_i (1 \le i \le n)$ is

(1) n (2) n – 1

(3) $\le$ n – 1 (4) $\le$ n

[June 2013 (3 Marks)]

3. Let $P : \mathbb{R} \to \mathbb{R}$ be a polynomial of the form $P(x) = a_0 + a_1 x + a_2 x^2$, with $a_0, a_1, a_2 \in \mathbb{R}$ and $a_2 \neq 0$. Let $E_1 = \int_0^1 P(x)\,dx - \frac{1}{2}(P(0) + P(1))$

$E_2 = \int_0^1 P(x)dx - p\left(\frac{1}{2}\right)$

If $|x|$ is the absolute value of $x \in \mathbb{R}$, then

(1) $|E_1| > |E_2|$ (2) $|E_2| > |E_1|$

(3) $|E_2| = |E_1|$ (4) $|E_2| = 2|E_1|$

[Dec. 2014 (3 Marks)]

42. Let $f : \mathbb{R} \to \mathbb{R}$ be a polynomial of the form $f(x) = a_0 + a_1 x + a_2 x^2$ with $a_0, a_1, a_2 \in \mathbb{R}$ and $a_2 \neq 0$.

If $E_1 = \int_{-1}^1 f(x)dx - [f(-1) + f(1)]$

$E_2 = \int_{-1}^1 f(x)\,dx - \frac{1}{2}(f(-1) + 2f(0) + f(1))$

and |x| is the absolute value of $x \in \mathbb{R}$, then

(1) $|E_1| < |E_2|$ (2) $|E_1| = 2|E_2|$

(3) $|E_1| = 4|E_2|$ (4) $|E_1| = 8|E_2|$

[June 2015 (3 Marks)]

43. Let $f(x)$ be a polynomial of unknown degree taking the values

x	0	1	2	3
$f(x)$	2	7	13	16

All the fourth divided differences are $-1/6$. Then the coefficient of x^3 is

(1) 1/3 (2) $-2/3$

(3) 16 (4) -1

[Dec. 2018 (3 Marks)]

97. Consider the interpolation data given below

x	1	1/2	3
y	3	-10	2

The interpolating polynomial corresponding to this data is given by

(1) $p(x) = -3\left(x - \frac{1}{2}\right)(x-3) - 8(x-1)(x-3) + \frac{2}{5}(x-1)\left(x - \frac{1}{2}\right)$

(2) $q(x) = 3 + 26(x-1) + \frac{-53}{5}(x-1)\left(x - \frac{1}{2}\right)$

(3) $r(x) = \frac{-53}{5}x^2 + \frac{419}{10}x + \frac{-283}{10}$

(4) p(x) q(x) + r(x)

[Dec. 2011 (4.75 Marks)]

101. To compute the value of e^t in the interval [0, 1], pick $t_1 = 0, t_2 = 0.5$ and $t_3 = 1$. Let p be the quadratic polynomial that interpolates e^t, that is, $p(t_i) = e^{t_i}$, i = 1, 2, 3. Then

(1) The polynomial p can be written in the form $L_1(t) + e^{1/2}L_2(t) + eL_3(t)$ for some choice of quadratic polynomials L_1, L_2, L_3.

(2) If the polynomial p is written in the form $L_1(t) + e^{1/2}L_2(t) + eL_3(t)$, where L_1, L_2 and L_3 are polynomials, then L_1, L_2 and L_3 are uniquely determined.

(3) If p is written in the form $L_1(t) + e^{1/2}L_2(t) + eL_3(t)$, then one of L_1, L_2 or L_3 must be linear

(4) The polynomial p is uniquely determined.

[Dec. 2012 (4.75 Marks)]

92. Let H(x) be the cubic Hermite interpolation of $f(x)=x^4+1$ on the interval $I=[0,1]$ interpolation at $x=0$ and $x=1$. Then

(1) can never be true

(1) $\max_{x\in I}|f(x)-H(x)|=\frac{1}{16}$

(2) The maximum of $|f(x)-H(x)|=\frac{1}{16}$

(3) $\max_{x\in I}|f(x)-H(x)|=\frac{1}{21}$

(4) The maximum of $|f(x)-H(x)|=\frac{1}{21}$ is attained at $x=\frac{1}{4}$.

[June 2016 (4.75 Marks)]

Qus 95.

Consider $\alpha=\frac{1+\sqrt{5}}{2}$ and $\beta=\frac{1-\sqrt{5}}{2}$. Define a sequence of numbers F_n as follows:

$$F_n=\frac{\alpha^n-\beta^n}{\alpha-\beta} \text{ for } n=1,2,\dots$$

Let $p:\mathbb{R}\to\mathbb{R}$ be a polynomial of degree at most 2 such that $p(1)=F_1$, $p(3)=F_3$, $p(5)=F_5$

Which of the following statements are true ?

(a) $F_n=F_{n-1}+F_{n-2}$ for $n\geq 3$

(b) $p(7)=13$

(c) $F_n=F_{n-1}+2F_{n-2}$ for $n\geq 5$

(d) $p(7)=10$

[16 sep 2022 (4.75 Marks)]

Answer Key

Chapter: - 3. Interpolation and Approximation:-

1.	**June 2013**	**Q. 26**	**(3)**
2.	**Dec. 2014**	**Q. 43**	**(1)**
3.	**June 2015**	**Q. 42**	**(3)**
4.	**Dec. 2018**	**Q. 43**	**(2)**
5.	**Dec. 2011**	**Q. 97**	**(1)**
6.	**Dec. 2012**	**Q. 101**	**(1,2,3)**
7.	**June 2016**	**Q. 92**	**(1,2)**
8.	**16 sep 2022**	**Q. 95**	**(1,4)**

Chapter:- 4

Ordinary Differential Equation Initial Value Problems:-

45. Consider a second order ordinary differential Equation (ODE) and its finite difference representation. Identify which of the following statements is correct.

(1) The finite difference representation is unique

(2) The finite difference representation is unique for some ODE

(3) There is no unique finite difference scheme for the ODE

(4) The uniqueness of a finite difference scheme can not be determined

[June 2012 (3 Marks)]

48. Consider the initial value problem

$\frac{dy}{dx} = x + y, y(0) = 1.$

Then the approximate value of the solution y(x) at x = 0.2, using improved Euler method, with h =0.2 is

(1) 1.11 (2) 1.20

(3) 1.24 (4) 1.48

[Dec. 2013 (3 Marks)]

96. Consider the ordinary differential equation

$\frac{dy}{dt} = \lambda y; \quad t > 0 \quad , \quad y(0) = 1$

and the Euler scheme with step size h

$\frac{Y_{n+1} - Y_n}{h} = \lambda Y_n; n \geq 1 \quad , \quad Y_0 = 1$

Which of the following are necessarily true for Y_1 which approximates $Y(h) = e^{\lambda h}$?

(1) Y_1 is a polynomial approximation

(2) Y_1 is a rational function approximation

(3) Y_1 is trigonometric function approximation

(4) Y_1 is a truncation of infinite series

[Dec. 2012 (4.75 Marks)]

116. A Runge-kutta method for numerically solving the initial-value ordinary differential equation

$y' = f(x,y); y(x_0) = y_0$ is given by (for h small)

$y(x + h) = y(x) + w_1F_1(x, y) + w_2F_2(x, y)$

$F_1(x, y) = hf(x, y), \; F_2(x, y) = hf(x + ah, y + \beta F_1)$

The objective is to determine the constants w_1, w_2, α and β such that the above formula is accurate to order 2 (that is, the error term is) (h^3)). Which of the following are correct sets of values for these constants?

(1) $w_1 = 1/2, w_2 = 1/2; \alpha = 1, \beta = 1$

(2) $w_1 = 2, w_2 = 1; \alpha = 1/2, \beta = 1/2$

(3) $w_1 = 1/3, w_2 = 2/3; \alpha = 3/4, \beta = 3/4$

(4) $w_1 = 3/4, w_2 = 1/4; \alpha = 2, \beta = 2$

[June 2013 (4.75 Marks)]

95. Let $y(t)$ satisfy the differential equation $y' = \lambda y; y(0) = 1$. Then the backward Euler method, for $n \geq 1$ and $h > 0$ $\frac{y_n - y_{n-1}}{h} = \lambda y_n; y_0 = 1$ *yields*

(1) a first order approximation to $e^{\lambda nh}$

(2) a polynomial approximation to $e^{\lambda nh}$

(3) a rational function approximation to $e^{\lambda nh}$

(4) a Chebyshev polynomial approximation to $e^{\lambda nh}$

[Dec. 2014 (4.75 Marks)]

98. Consider the Runge-Kutta method of the form

$y_{n+1} = y_n + ak_1 + bk_2$

$k_1 = hf(x_n, y_n)$

$k_2 = hf(x_n + ah, y_n + \beta k_1)$ to approximate the solution of the initial value problem

$y'(x) = f(x, y(x)), y(x_0) = y_0.$

Which of the following choice of a,b, α and β yield a second order method?

(1) $a = \frac{1}{2}, b = \frac{1}{2}, a = 1, \beta = 1$

(2) $a = 1, b =, a = \frac{1}{2}, \beta = \frac{1}{2}$

(3) $a=\frac{1}{4}, b=\frac{3}{4}, \alpha=\frac{2}{3}, \beta=\frac{2}{3}$

(4) $a=\frac{3}{4}, b=\frac{1}{4}, \alpha=1\beta=1$

[June 2016 (4.75 Marks)]

98. The order of linear multi step method

$u_{j+1}=(1-a)u_j+au_{j-1}+\frac{h}{4}\{(a+3)u'_{j+1}+(3a+1)u'_{j-1}\}$

for solving $u' f(x,u)$ is

(1) 2 if $a=-1$ (2) 2 if $a=-2$

(3) 3 if $a=-1$ (4) 3 if $a=-2$

[Dec. 2016 (4.75 Marks)]

95. For a differentiable function $f:\mathbb{R}\to\mathbb{R}$ define the difference quotient

$$(D_x f)(h)=\frac{f(x+h)-f(x)}{h}; h>0.$$

Consider numbers of the form $h=h(1+\varepsilon)$ for a fixed $\in>0$ and let

$e_1(h)=f'(x)-(D_x f)(h),$

$e_2(h)=(D_x f)(h)-(D_x f)(\hat{h})$

$e(h)=e_1(h)+e_2(h)$

If $f(x+\hat{h})=f(x+h)$, then

(1) $e_1(h)\to$ as $h\to 0.$

(2) $e_2(h)\to 0$ as $h\to 0.$

(3) $e_2(h)\to\in f'(x)/(1+\in).$ as $h\to 0.$

(4) $e(h)\to 0$ as $h\to 0.$

[June 2017 (4.75 Marks)]

96. Let y_n satisfy $y_n=y_{n-1}+hy_{n-1}$ with $y_0=1(n=1,2,..,N)$ and for $0<h<1, Nh=1$. Then

(1) $y_N\to e$ as $N\to\infty$

(2) $y_N\to e^h$ as $N\to\infty$

(3) $y_n=(1+h)^n$

(4) $y_n\geq 1$

[June 2017 (4.75 Marks)]

97. The forward difference operator is defined as $\vartriangle U_n-3\Delta U_n+2U_n=0$. Then which of the following difference equations has an unbounded general solution?

(1) $\Delta^2 U_n-3\Delta U_n+2U_n=0$

(2) $\Delta^2 U_n+\Delta U_n+\frac{1}{4}U_n=0$

(3) $\Delta^2 U_n-2\Delta U_n+2U_n=0$

(4) $\Delta^2 U_{n+1}-\frac{1}{3}\Delta^2 U_n=0$

[June 2018 (4.75 Marks)]

93. Let $u(x)$ satisfy the boundary value problem

$$(BVP)\begin{cases} u''+u'=0,\ x\in(0,1) \\ u(0)=0 \\ u(1)=1. \end{cases}$$

Consider the finite difference approximation to (BVP)

$$(BVP)_h\begin{cases} \frac{U_{j+1}-2U_j+U_{j-1}}{h^2}+\frac{U_{j+1}-U_{j-1}}{2h}=0, j=1,\dots,N-1 \\ U_0=0 \\ U_N=1 \end{cases}$$

Here U_j is an approximation to $u(x_j)$ where $x_j=ih, j=0,\dots,N$ is a partition of [0,1] with $h=1/N$ for some positive integer N. Then which of the following are true?

(1) There exists a solution to $(BVP)_h$ of the form $U_j=ar^j+b$ for some $a,b\in\mathbb{R}$ with $r\neq 1$ and r satisfying $(2+h)r^2-4r+(2-h)=0$

(2) $U_j=(r^j-1)/(r^N-1)$ where r satisfies $(2+h)r^2-4r+(2-h)=0$ and $r\neq 1$

(3) u is monotonic in x

(4) U_j is monotonic in j.

[Dec. 2018 (4.75 Marks)]

Qus 97.

Consider the ODE $\dot{x} = f(t,x)$ in $\mathbb{R}$, for a smooth function f.

Consider a general second order Runge-Kutta formula of the form

$$x(t+h) = x(t) + w_1 h f(t,x) + w_2 h f(t+\alpha h, x + \beta h f) + O(h^3).$$

Which of the following choices of $(w_1, w_2, \alpha, \beta)$ are correct ?

(a)

$\left(\frac{1}{2}, \frac{1}{2}, 1, 1\right)$

(b)

$\left(\frac{1}{2}, 1, \frac{1}{2}, 1\right)$

(c)

$\left(\frac{1}{4}, \frac{3}{4}, \frac{2}{3}, \frac{2}{3}\right)$

(d)

$(0, 1, 1, 1)$

[16 sep 2022 (4.75 Marks)]

Answer Key

Chapter: - 4. Ordinary Differential Equation Initial Value Problem:-

1.	June 2012	Q. 45	(3)
2.	Dec. 2013	Q. 48	(3)
3.	Dec. 2012	Q. 96	(1,4)
4.	June 2013	Q. 116	(1,3,4)
5.	Dec. 2014	Q. 95	(1,3)
6.	June 2016	Q. 98	(1,3)
7.	Dec. 2016	Q. 98	(2,3)
8.	June 2017	Q. 95	(1,3)
9.	June 2017	Q. 96	(1,3,4)
10.	June 2018	Q. 97	(1,3,4)
11.	Dec. 2018	Q. 93	(1,2,3,4)
12.	16 sep 2022	Q. 97	(1,3)

Chapter:- 5

System of Linear Algebraic Equations and Eigenvalue Problems:-

43. Consider solving the following system by Jacobi iteration scheme

$x+2my-2mz=1$

$nx+y+nz=2$

$2mx+2my+z=1$

where $m,n\in\mathbb{Z}$. With any initial vector, the scheme converges provided m,n, satisfy

(1) $m+n=3$ (2) $m>n$
(2) $m<n$ (4) $m=n$

[June 2019 (3 Marks)]

97. Consider a linear system Ax = b with a computed solution x_c; the error and the residue are defined, respectively by

$e = x - x_c$

$r = Ax - Ax_c$

Then

(1) A small error necessarily implies a small residue.
(2) The error can be large with relatively small residue.
(3) The error can be small with relatively large residue.
(4) The error and the residue are always equal.

[June 2011 (4.75 Marks)]

99. Let $A=\begin{pmatrix}1.01 & 0.99\\ 0.99 & 1.01\end{pmatrix}, b=\begin{pmatrix}2\\2\end{pmatrix}$ and x_0 be the unique solution of the equation Ax = b.

Let $\hat{x}_1$ and $\hat{x}_2$ be the two approximate solutions $\begin{pmatrix}1.01\\1.01\end{pmatrix}$ and $\begin{pmatrix}2\\0\end{pmatrix}$. Finally, let $r_1=Ax_{.0}-A\hat{x}_1$ $=b-A\hat{x}_1$ and $r_2=Ax_{.0}-A\hat{x}_2=b-A\hat{x}_2$ be the corresponding residues.

Which of the following is/are correct?

(1) $\hat{x}_1$ is a good approximation to $\hat{x}_0$ and r_1 is small
(2) $\hat{x}_1$ is not a good approximation to $\hat{x}_0$ but r_1 is small
(3) $\hat{x}_2$ is a good approximation to x_0 and r_2 is small
(4) $\hat{x}_2$ is not a good approximation to x_0 but r_2 is small

[Dec. 2011 (4.75 Marks)]

97. Given that an upper triangular matrix (UTM) is invertible if and only if all its diagonal elements are different from zero, consider the linear system

$$\begin{aligned}2x_1+3x_2-x_3&=5\\4x_1+4x_2-3x_3&=3\\-2x_1+3x_2-x_3&=1\end{aligned}\qquad\ldots(1)$$

Then system (1)

(1) Can be transformed into an UTM but is not invertible because the diagonal entries of the UTM are non different from zero.
(2) Is invertible though cannot be transformed into an UTM
(3) Can be transformed into an UTM because above diagonal entries are all different from zero.
(4) Can be transformed into an UTM and the solution of the UTM is the solution of (1).

[June 2012 (4.75 Marks)]

96. Consider the linear system $Ax=b$ with $A=\begin{bmatrix}2 & 1 & -3\\ 1 & 2 & -2\\ -3 & -2 & 1\end{bmatrix}$ Let x_n denote the n^{th} Gauss-Seidel iteration and $e_n=x_n-x$ .Let M be the corresponding matrix such that $e_{n+1}=Me_n, n\geq 0$. Which of the following statments are necessarily true?

(1) all eigenvalues of M have absolute value less than 1
(2) there is an eigenvalues of M with absolute value at least 1

(3) e_n converges to 0 as $n \to \infty$ for all $b \in \mathbb{R}^3$ and any e_0

(4) e_n does not converge to 0 as $n \to \infty$ for any $b \in \mathbb{R}^3$ unless $e_0 = 0$

[Dec. 2017 (4.75 Marks)]

96. Assume that a non-singular matrix $A = L + D + U$ where L and U are lower and Upper triagular matrices respectively with all diagonal entries are zero and D is a diagonal matrix. Let x be the solultion of $Ax = b.$ Then the Gauss-Seidel ieration method $x^{(k+1)} = Hx^{(k)} + c, \quad k = 0,1,2,...$ with $\|H\| < 1$ converges to x* provided H is equal to

(1) $-D^{-1}(L+U)$ (b) $-(D+L)^{-1}U$

(3) $-D(L+U)^{-1}$ (d) $-(L-D)^{-1}U$

[June 2018 (4.75 Marks)]

Qus 47.
Let A be the following invertible matrix with real positive entries:

$$A = \begin{pmatrix} 1 & 2 \\ 8 & 9 \end{pmatrix}$$

Let G be the associated Gauss-Seidel iteration matrix. What are the two eigenvalues of G ?

(a) 0 and $4/3$

(b) 0 and $-4/3$

(c) 0 and $16/9$

(d) $4/3$ and $-4/3$

[16 sep 2022 (3 Marks)]

Answer Key

Chapter: - 5. System of Linear Algebraic Equations and Eigenvalue Problems:-

1.	**June 2019**	**Q. 43**	**(4)**
2.	**June 2011**	**Q. 97**	**(2,3)**
3.	**Dec. 2011**	**Q. 99**	**(1,4)**
4.	**June 2012**	**Q. 97**	**(3,4)**
5.	**Dec. 2017**	**Q. 96**	**(2,4)**
6.	**June 2018**	**Q. 96**	**(2)**
7.	**16 sep. 2022**	**Q. 47**	**(3)**

Chapter:- 6

Differentiation and Integration:-

45. The values of a, b, c such that

$$\int_0^h f(x)dx = h\left\{af(0) + bf\left(\frac{h}{3}\right) + cf(h)\right\}$$ is exact

for polynomias f of degree as high as possible are

(1) $a = 0, b = \frac{3}{4}, c = \frac{1}{4}$

(2) $a = \frac{3}{4}, b = \frac{2}{4}, c = \frac{1}{4}$

(3) $a = \frac{-2}{4}, b = \frac{3}{4}, c = \frac{1}{4}$

(4) $a = 0, b = \frac{1}{4}, c = \frac{3}{4}$

[June 2018 (3 Marks)]

101. The following numerical integration formula is exact for all polynomials of degree less than or equal to 3

(1) Trapezoidal rule

(2) Simpson's $\frac{1}{3}rd$ rule

(3) Simpson's $\frac{3}{8}$th rule

(4) Gauss-Legendre 2 point formula

[June 2015 (4.75 Marks)]

101. Let $f:[0,3] \to \mathbb{R}$ be defined by $f(x) = |1 - |x-2||$ where $|.|$ denoteds the absolute value. Then for the numerical approximation of $\int_0^3 f(x)dx$, which of the following statements are true?

(1) The composite trapezoid rule with three equal subinterval is exact.

(2) The composite midpoint rule with three equal subintervals is exact

(3) The composite trapezoid rule with four equal subintervals in exact

(4) The composite midpoint rule with four equal
subintervals is exact.

[June 2016 (4.75 Marks)]

97. For $f \in C[0,1]$ and $n > 1$, let

$$T(f) = \frac{1}{n}\left[\frac{1}{2}f(0) + \frac{1}{2}f(1) + \sum_{j=1}^{n-1} f\left(\frac{j}{n}\right)\right]$$ be an

approximation of the integral $I(f) = \int_0^1 f(x)dx$.

For which of the following function f is $T(f) = I(f)$?

(1) $1 + \sin 2\pi nx$

(2) $1 + \cos 2\pi nx$

(3) $\sin^2 2\pi nx$

(4) $\cos^2 2\pi(n+1)x$

[Dec. 2017 (4.75 Marks)]

96. The values of α, A, B, C for which the quadrature formula

$$\int_{-1}^{1}(1-x)f(x)dx = Af(-\alpha) + Bf(0) + cf(\alpha)$$

is exact for polynomials of highest possible degree are

(1) $\alpha = \sqrt{\frac{3}{5}}, A = \frac{5}{9} + \frac{\sqrt{5}}{3\sqrt{3}}, B = \frac{8}{9}, C = \frac{5}{9} - \frac{\sqrt{5}}{3\sqrt{3}}$

(2) $\alpha = \sqrt{\frac{3}{5}}, A = \frac{5}{9} - \frac{\sqrt{5}}{3\sqrt{3}}, B = \frac{8}{9}, C = \frac{5}{9} + \frac{\sqrt{5}}{3\sqrt{3}}$

(3) $\alpha = \sqrt{\frac{3}{5}}, A = \frac{5}{9}\left(1 - \frac{\sqrt{3}}{\sqrt{5}}\right) B = \frac{8}{9}, C = \frac{5}{9}\left(1 + \frac{\sqrt{3}}{\sqrt{5}}\right)$

(4) $\alpha = \sqrt{\frac{3}{5}}, A = \frac{5}{9}\left(1 + \frac{\sqrt{3}}{\sqrt{5}}\right) B = \frac{8}{9}, C = \frac{5}{9}\left(1 - \frac{\sqrt{3}}{\sqrt{5}}\right)$

[Dec. 2019 (4.75 Marks)]

100. Consider the ordinary differential equation (ODE)

$$\begin{cases} y'(x)+y(x)=0, & x>0 \\ y(1)=1 \end{cases}$$

and the following numerical scheme to solve the ODE

$$\begin{cases} \dfrac{Y_{n+1}-Y_{n-1}}{2h}+Y_{n-1}=0, & n\geq 1 \\ Y_0=1,\ Y_1=1 \end{cases}$$

If $0<h<\dfrac{1}{2}$, then which of the following statements are true ?

(1) $(Y_n)\to\infty$ as $n\to\infty$

(2) $(Y_n)\to 0$ as $n\to\infty$

(3) (Y_n) is bounded

(4) $\max\limits_{nh\in[0,T]}\left|y(nh)-Y_n\right|\to\infty$ as $T\to\infty$

[Dec. 2019 (4.75 Marks)]

Answer Key

Chapter: - 6. Differentiation and Integration:-

1.	**June 2018**	**Q. 45**	**(1)**
2.	**Dec. 2013**	**Q. 98**	**(3)**
3.	**June 2015**	**Q. 101**	**(2,3,4)**
4.	**June 2016**	**Q. 101**	**(1,4)**
5.	**Dec. 2017**	**Q. 97**	**(1)**
6.	**Dec. 2019**	**Q. 96**	**(1,4)**
7.	**Dec. 2019**	**Q. 100**	**(2,3)**